18. Find two linearly independent solutions of the equation $(3x-1)^2 y'' + (9x-3)y' - 9y = 0$ for $x > \frac{1}{3}$.

19. Find the indicial polynomial, and its roots, corresponding to the point $x = 1$.

20. State and prove the existence theorem the convergence of the successive approximation.

NOVEMBER/DECEMBER 2023

23PMA13 — ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Give the example of initial value problem.

2. Write a formula for the Wronskian.

3. Give the example of non-homogeneous equations of order 2.

4. What is annihilator method?

5. State existence and uniqueness theorem.

6. Write Legendre- Equation.

7. State Euler equation.

8. Give Bessel equation.

9. Write Lipschitz condition.

10. Write the method of successive approximations.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Derive the formula for the Wronskian.

Or

(b) Solve $y'' - 2y' - 3y = 0$, $y(0) = 0$, $y'(0) = 1$.

12. (a) Let $\phi_1, \ldots \phi_n$ be n linearly independent solutions of $L(y) = 0$ on an interval I. If $c_i \ldots, c_n$ are any constants $\phi = c_1\phi_1 + \ldots + c_n\phi_n$ is a solution and every solution may be represented in this form.

Or

(b) Find all the solution of $y''' - 3iy'' - 3y' + iy = 0$.

13. (a) One solution of $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

Or

(b) Show that $W(\phi_1, \phi_2)(x) = 1$ for all x.

14. (a) Find all solutions of the following equations for $x > 0$, $xy'' + xy' - 4y = x$.

Or

(b) Solve $xy' + xy - 6y = x^2$.

15. (a) Find a sequence of successive approximations for the problem $y'' - x - y$, $y(0) = 1$, $y'(0) = 0$, show that the sequence tends to a limit for all real x.

Or

(b) Find the real valued solution of the equation $y' = \frac{e^x - y}{1 + e^x}$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that for any real x_0, and constants α, β, there exists a solution ϕ, of the initial value problem $L(y) = 0$, $y(x_0) = \alpha$, $y'(x_0) = \beta$ on $-\infty < x < \infty$.

17. Find all real-valued solutions of the following equations:

(a) $y'' + y = 0$ (b) $y'' - y = 0$